

Wave potential: a unified model of arterial waves, reservoir phenomena and their interaction

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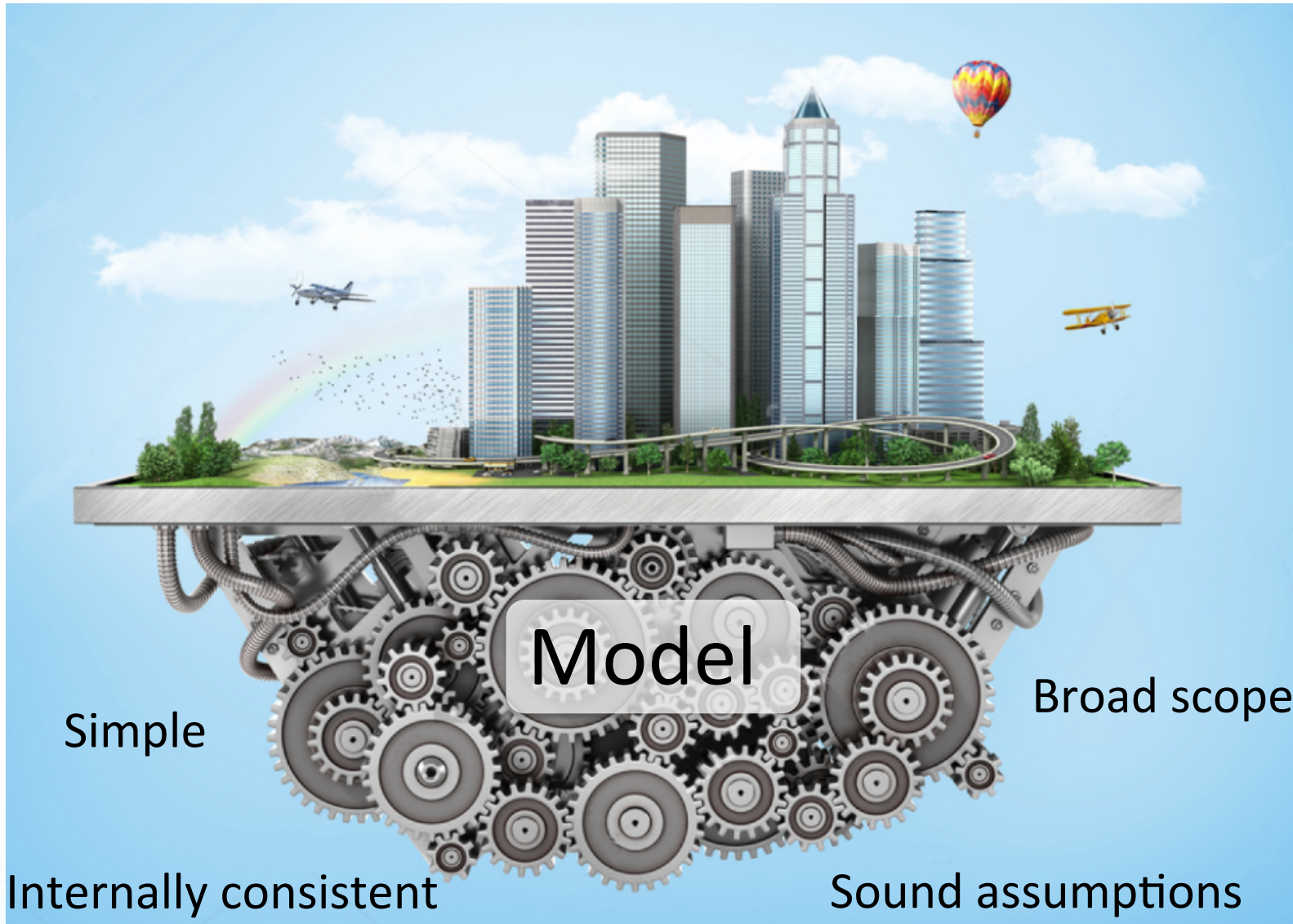
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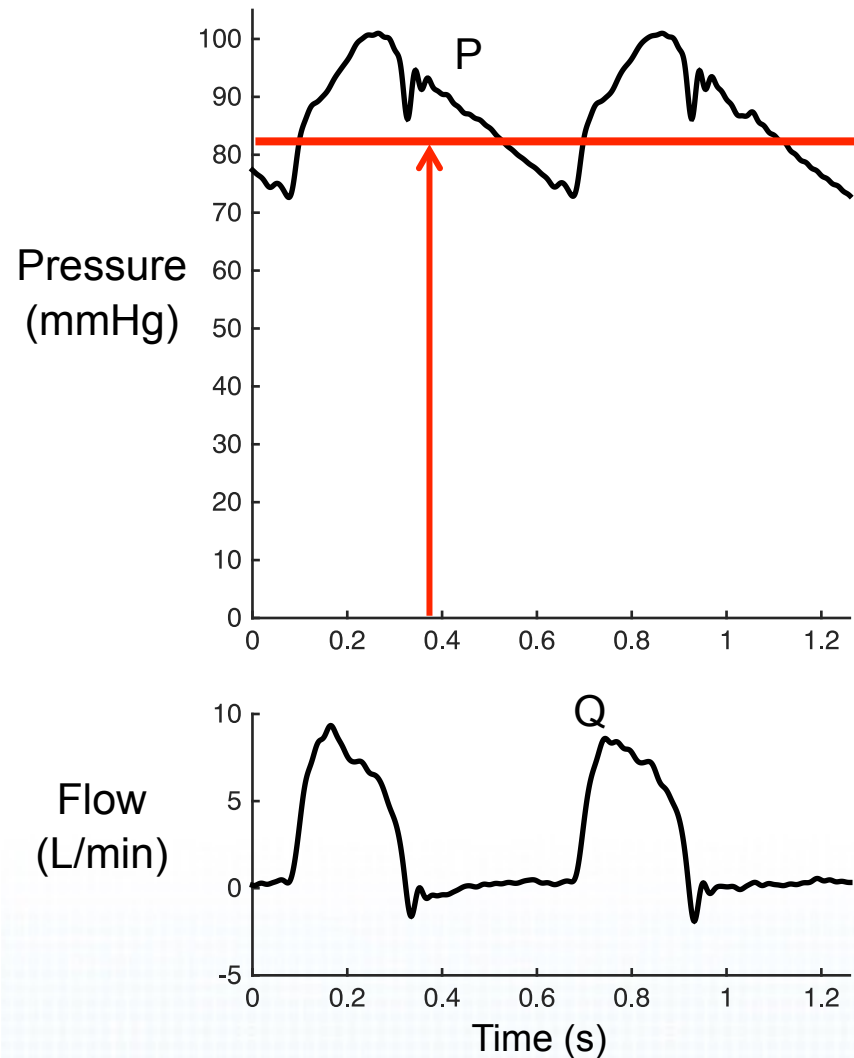
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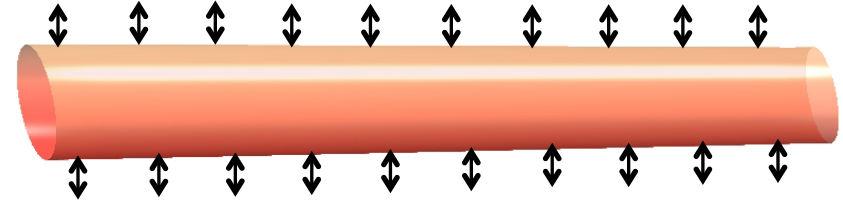


Wanted: a unified model of haemodynamics

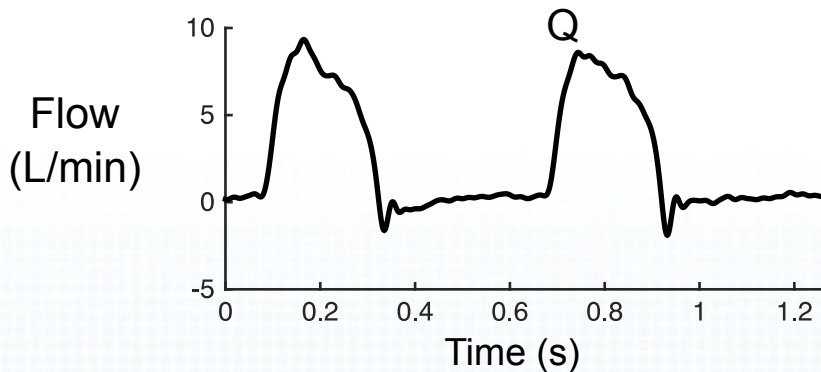
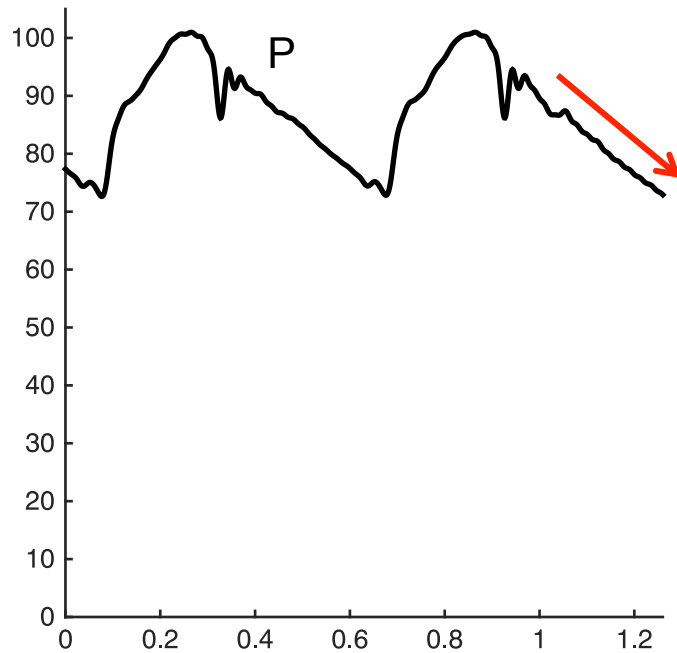


Mean pressure

Volume storage / discharge in large arteries



Wanted: a unified model of haemodynamics



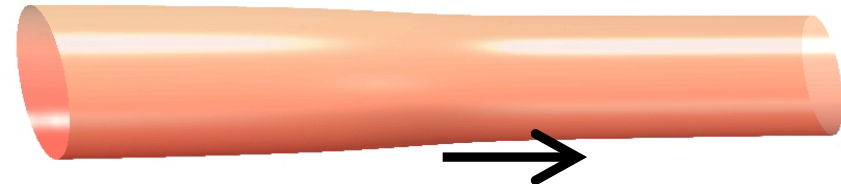
Mean pressure

Volume storage / discharge in large arteries

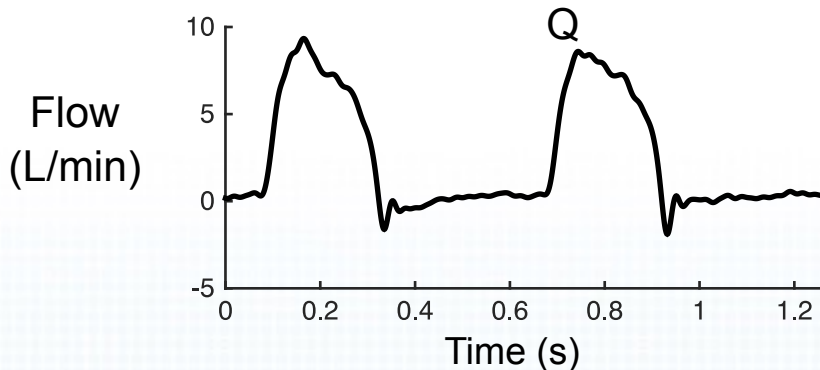
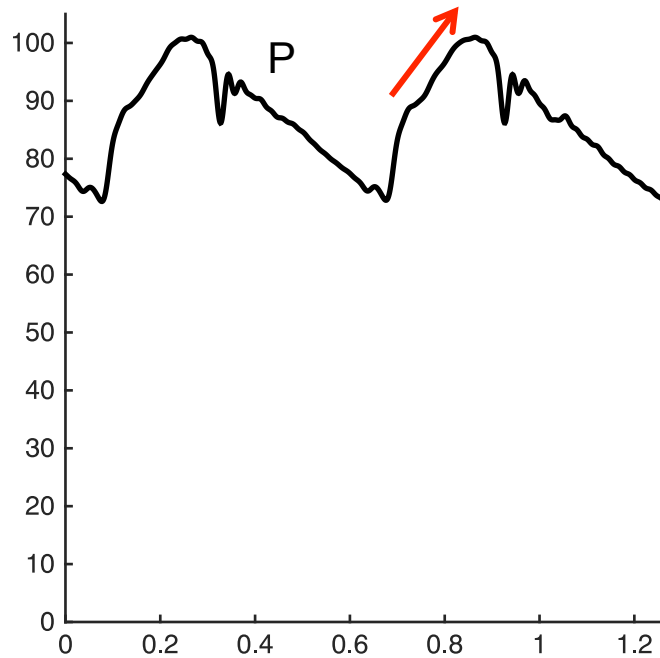


Exponential diastolic P decay

Pulse propagation (PWV) and reflection



Wanted: a unified model of haemodynamics



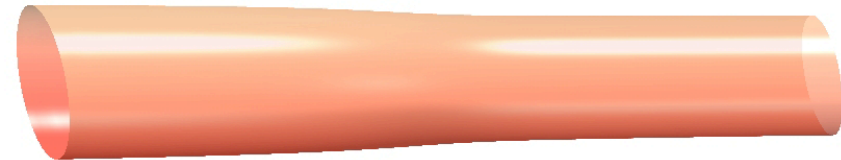
Mean pressure

Volume storage / discharge in large arteries



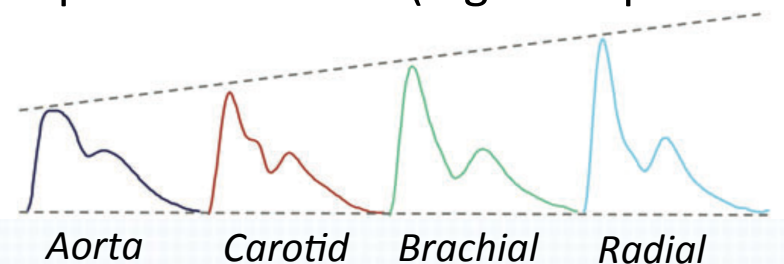
Exponential diastolic P decay

Pulse propagation (PWV) and reflection



P/Q pulse features

- Systolic P augmentation
- Spatial evolution (e.g. P amplification)



Wanted: a unified model of haemodynamics

	Windkessel	Wave Model
Mean Pressure	✓	✗
Volume storage	✓	✗
Exponential P decay	✓	??
Pulse propagation	✗	✓
P Augmentation	✗	✓
P Amplification	✗	✓



Assumes infinite
wave speed

Reservoir-Wave Model

$$P(x,t) = P_{\text{res}}(t) + P_{\text{ex}}(x,t)$$

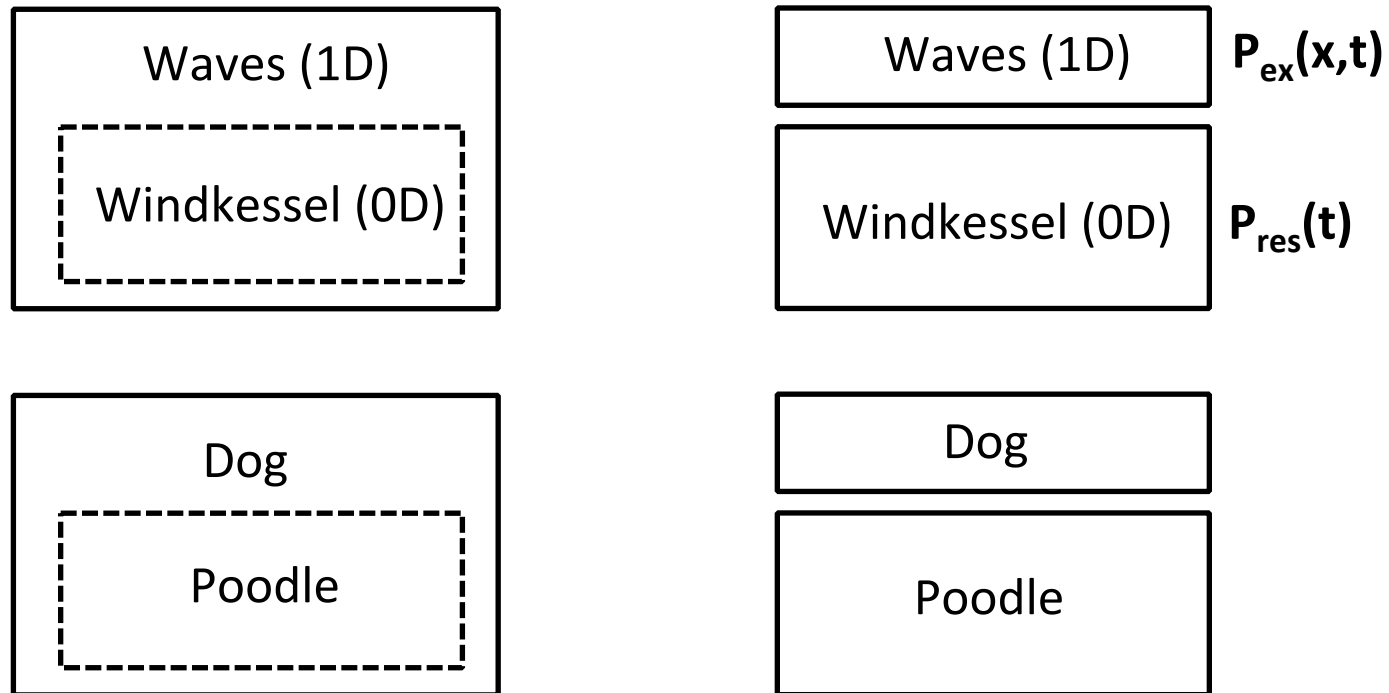
	Windkessel	Wave Model	Reservoir-Wave
Mean Pressure	✓	✗	✓
Volume storage	✓	✗	✓
Exponential P decay	✓	??	✓
Pulse propagation	✗	✓	✓
P Augmentation	✗	✓	✓
P Amplification	✗	✓	✓

Diagrammatic annotations: A green oval encircles the 'Mean Pressure', 'Volume storage', and 'Exponential P decay' rows in the Windkessel column. A green arrow points from this oval to the 'Reservoir-Wave' column. Another green oval encircles the 'Pulse propagation', 'P Augmentation', and 'P Amplification' rows in the Wave Model column. A green arrow points from this oval to the 'Reservoir-Wave' column.

Wang JJ, O'Brien AB, Shrive NG, Parker KH, and Tyberg JV. Time-domain representation of ventricular-arterial coupling as a windkessel and wave system. *Am J Physiol Heart Circ Physiol* 284: H1358-H1368, 2003.

Reservoir-Wave Model

Mathematical difficulty for the goal of separating 0D & 1D effects



Mynard JP et al. *Journal of Hypertension*. 30:734-43, 2012

Mynard JP. *IEEE Engineering in Medicine and Biology Society*, Osaka, Japan, 2013, p. 213-216.

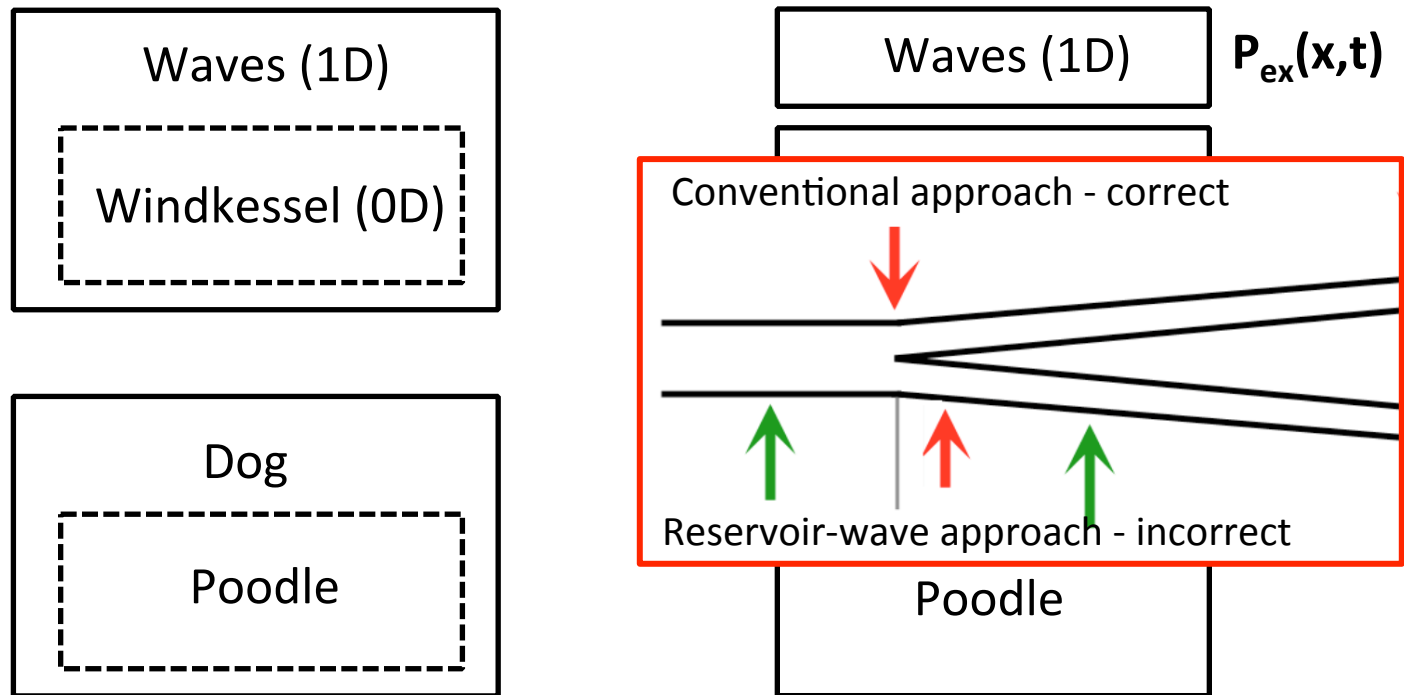
Mynard JP and Smolich JJ. *International Journal of Cardiology*. 176:1009-1012, 2014.

Mynard JP, Smolich JJ, Avolio A. *Journal of Hypertension*. 33:461-4, 2015.

Segers P et al. *Journal of Hypertension*. 33:554-563, 2015.

Reservoir-Wave Model

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Wanted: a unified model of haemodynamics

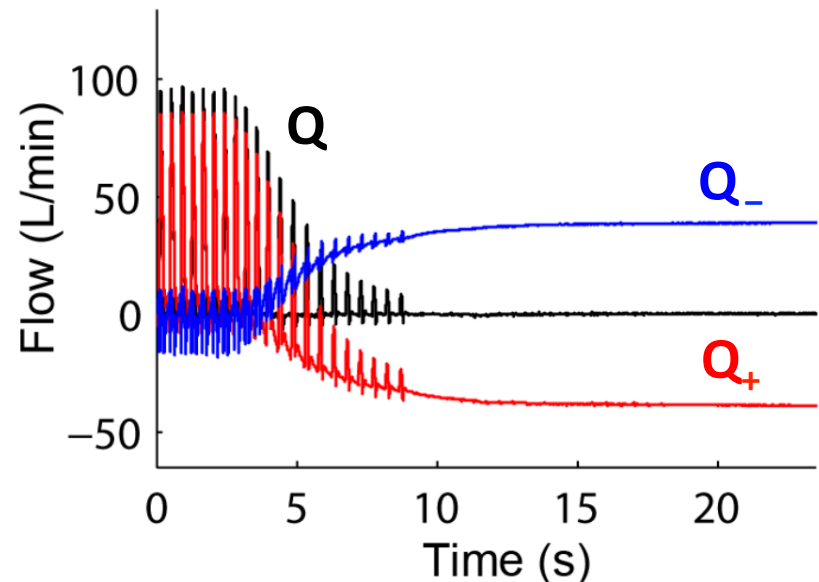
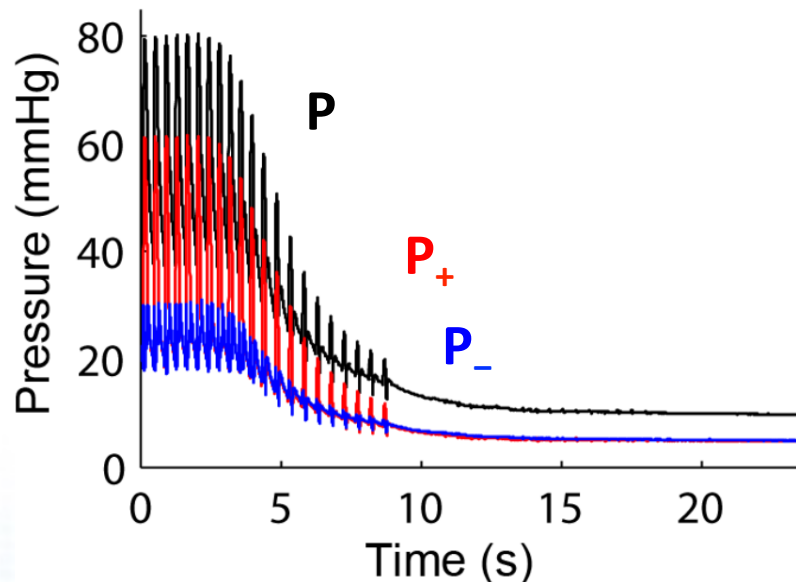
	Windkessel	Wave Model V1	Wave Model V2
Mean Pressure	✓	✗	💡
Volume storage	✓	✗	💡
Exponential P decay	✓	✗	💡
Pulse propagation	✗	✓	✓
P Augmentation	✗	✓	✓
P Amplification	✗	✓	✓

Conventional wave separation

With extended asystole (e.g. death), wave components do not fall to zero

- absolute values of P_{\pm} and Q_{\pm} are *arbitrary* (arbitrary constants c_p and c_Q)

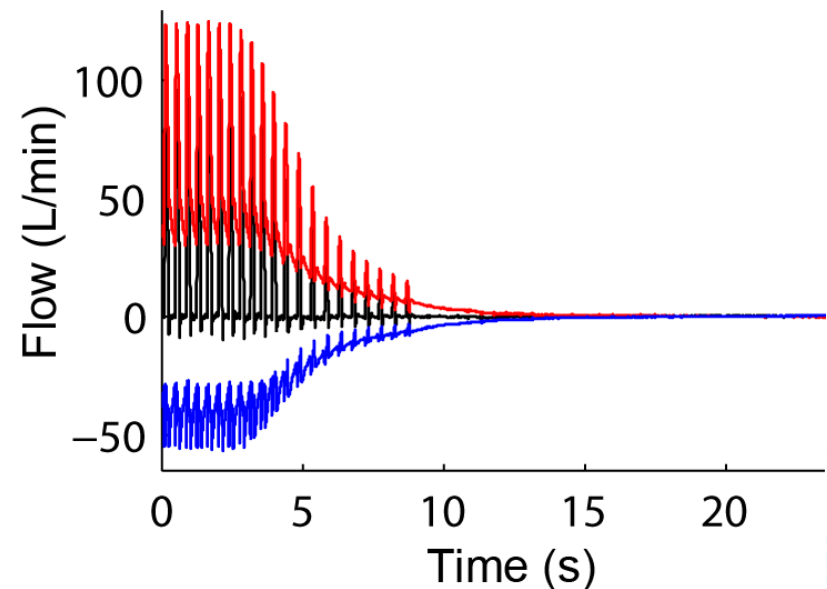
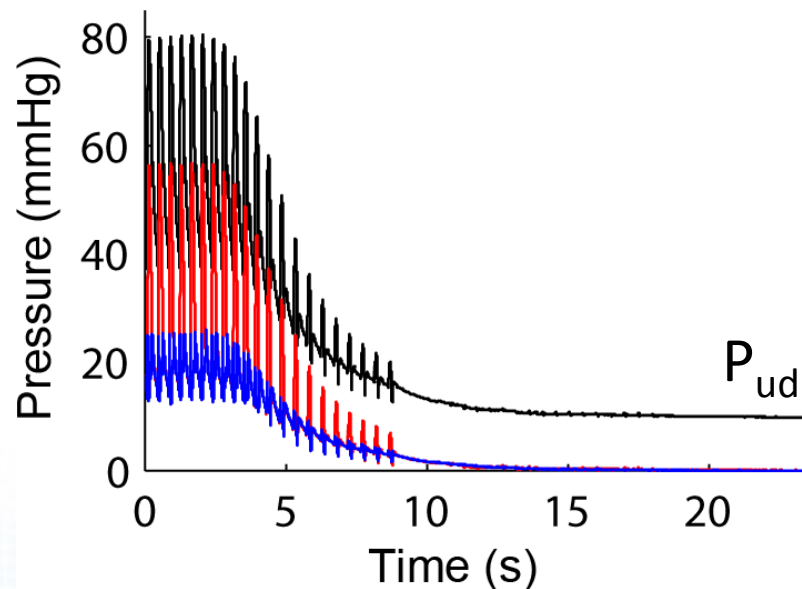
$$P_{\pm} = \frac{1}{2} \left(P \pm Z_c Q \right) + c_p \quad Q_{\pm} = \frac{1}{2} \left(Q \pm \frac{1}{Z_c} P \right) + c_Q$$



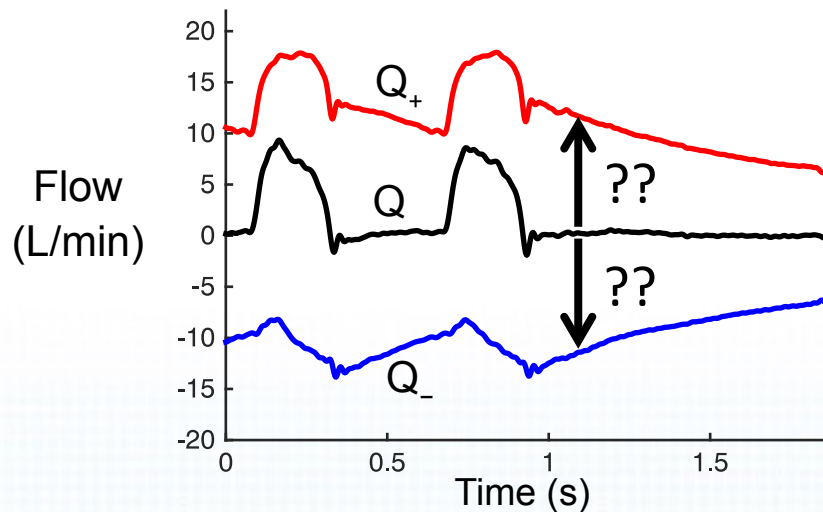
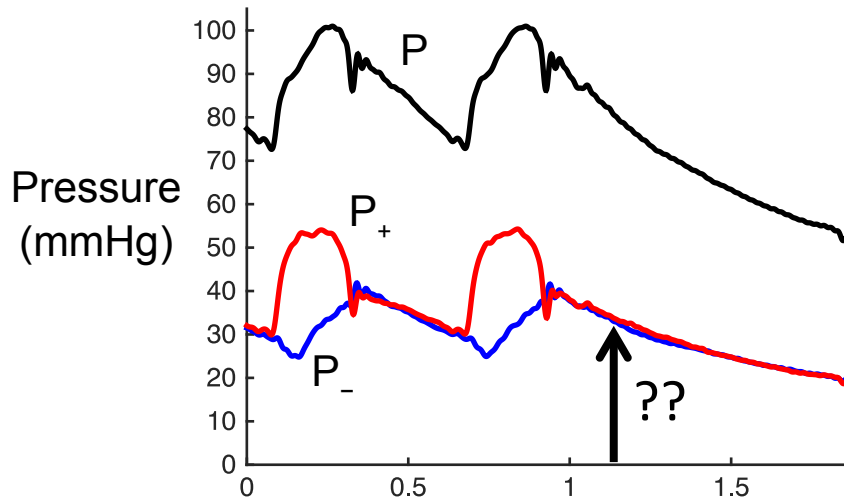
Wave separation V2

- Blood pressure converges to an 'undisturbed pressure' (P_{ud})
- New expressions for wave separation (non-arbitrary offset)

$$P_{\pm} = \frac{1}{2} \left(P - P_{ud} \pm Z_c Q \right) \quad Q_{\pm} = \frac{1}{2} \left(Q \pm \frac{1}{Z_c} (P - P_{ud}) \right)$$



Wave potential



What is the physical meaning of absolute P_{\pm} and Q_{\pm} ?

...

Wave potential!

Mynard JP and Smolich JJ. Wave potential and the one-dimensional windkessel as a wave-based paradigm of diastolic arterial hemodynamics. *Am J Physiol Heart Circ Physiol* 307: H307-H318, 2014.

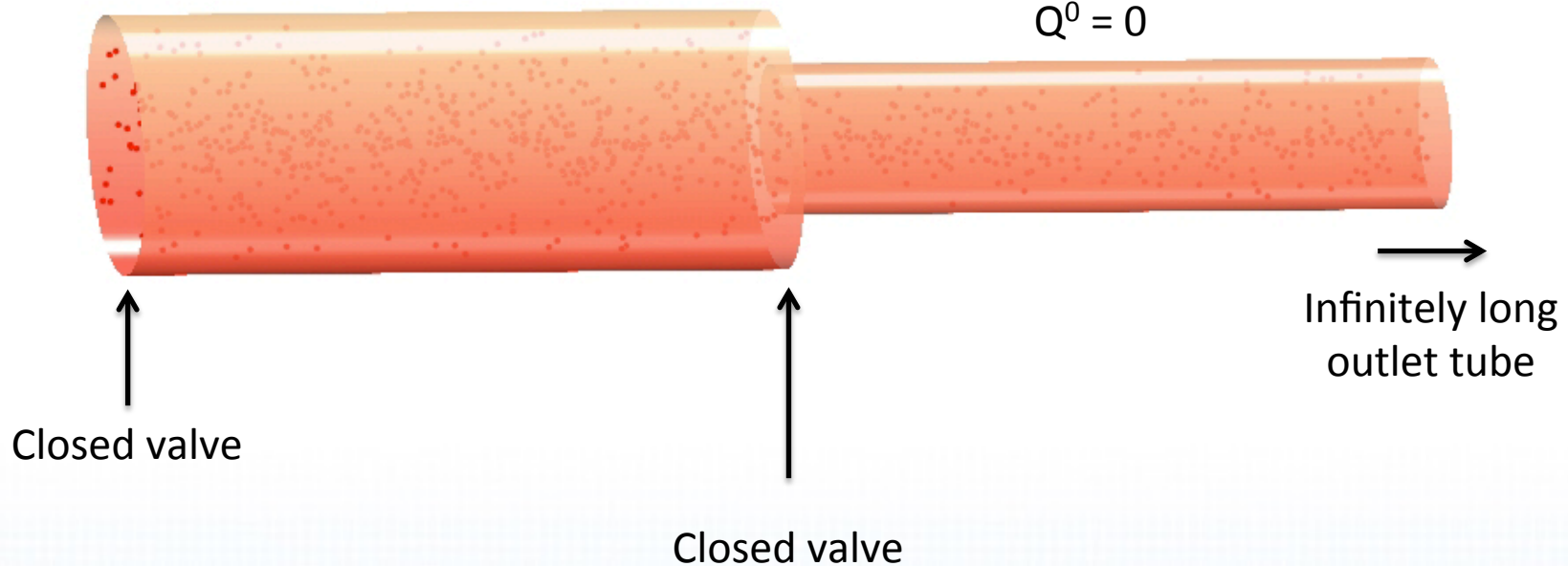
Wave potential and waves

$$P_{\pm} = 5 \text{ mmHg}$$
$$Q_{\pm} = \pm 78.6 \text{ mL/s}$$

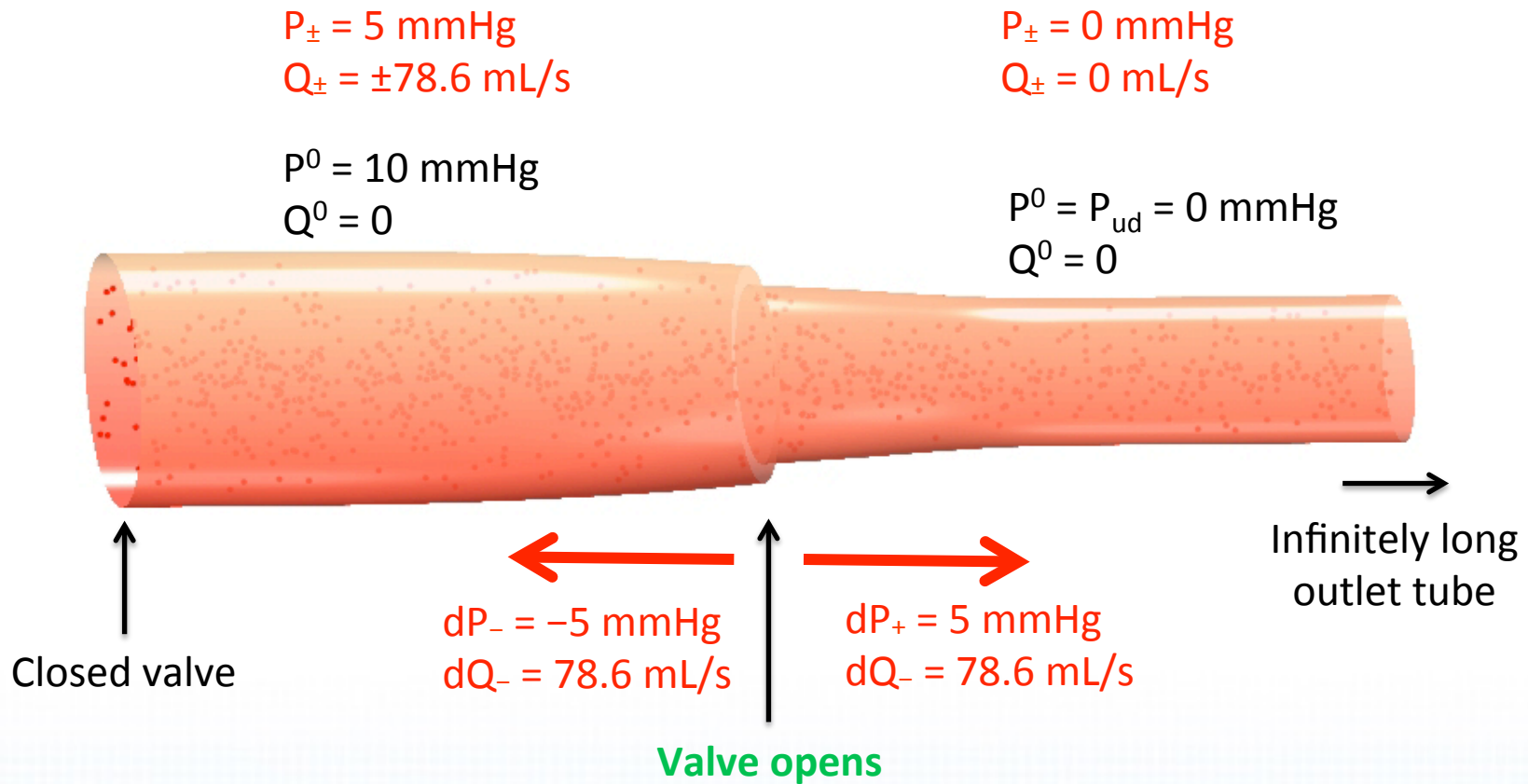
$$P^0 = 10 \text{ mmHg}$$
$$Q^0 = 0$$

$$P_{\pm} = 0 \text{ mmHg}$$
$$Q_{\pm} = 0 \text{ mL/s}$$

$$P^0 = P_{ud} = 0 \text{ mmHg}$$
$$Q^0 = 0$$

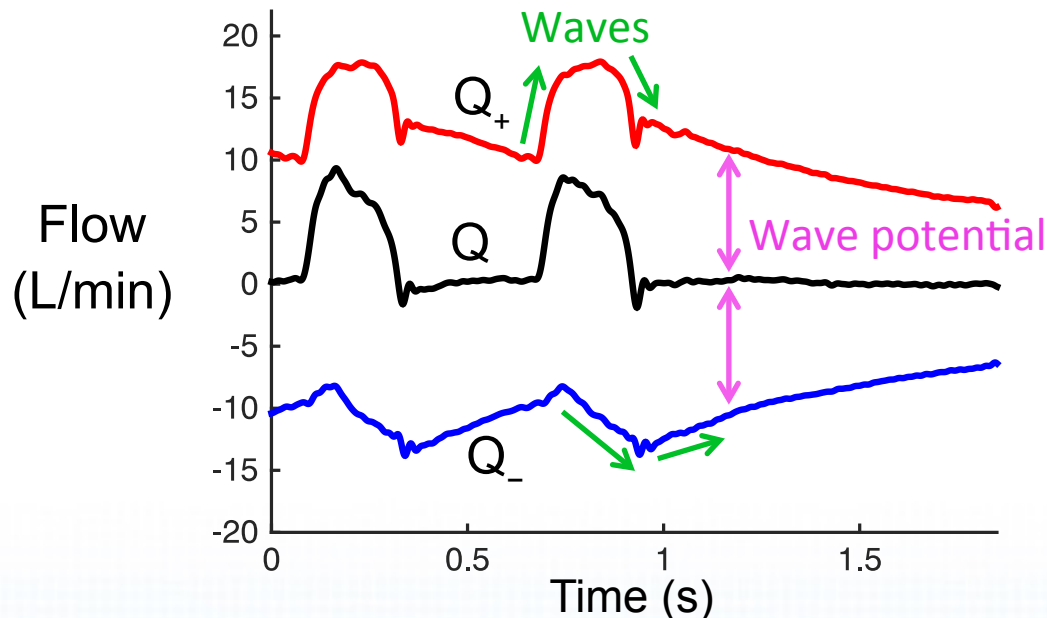


Wave potential and waves



Wave potential and waves

- **Wave potential** (absolute P_{\pm} & Q_{\pm}) represents the potential for wave generation
- Spatial gradients of P_{\pm} and Q_{\pm} produce propagating waves
- **Waves** cause spatio-temporal perturbations of P_{\pm} and Q_{\pm} (i.e. propagating dP_{\pm} and dQ_{\pm})



Wave potential and reservoir function

$$P_{\pm} = 5 \text{ mmHg}$$

$$Q_{\pm} = \pm 78.6 \text{ mL/s}$$

$$P^0 = 10 \text{ mmHg}$$

$$Q^0 = 0$$

$$P_{\pm} = 0 \text{ mmHg}$$

$$Q_{\pm} = 0 \text{ mL/s}$$

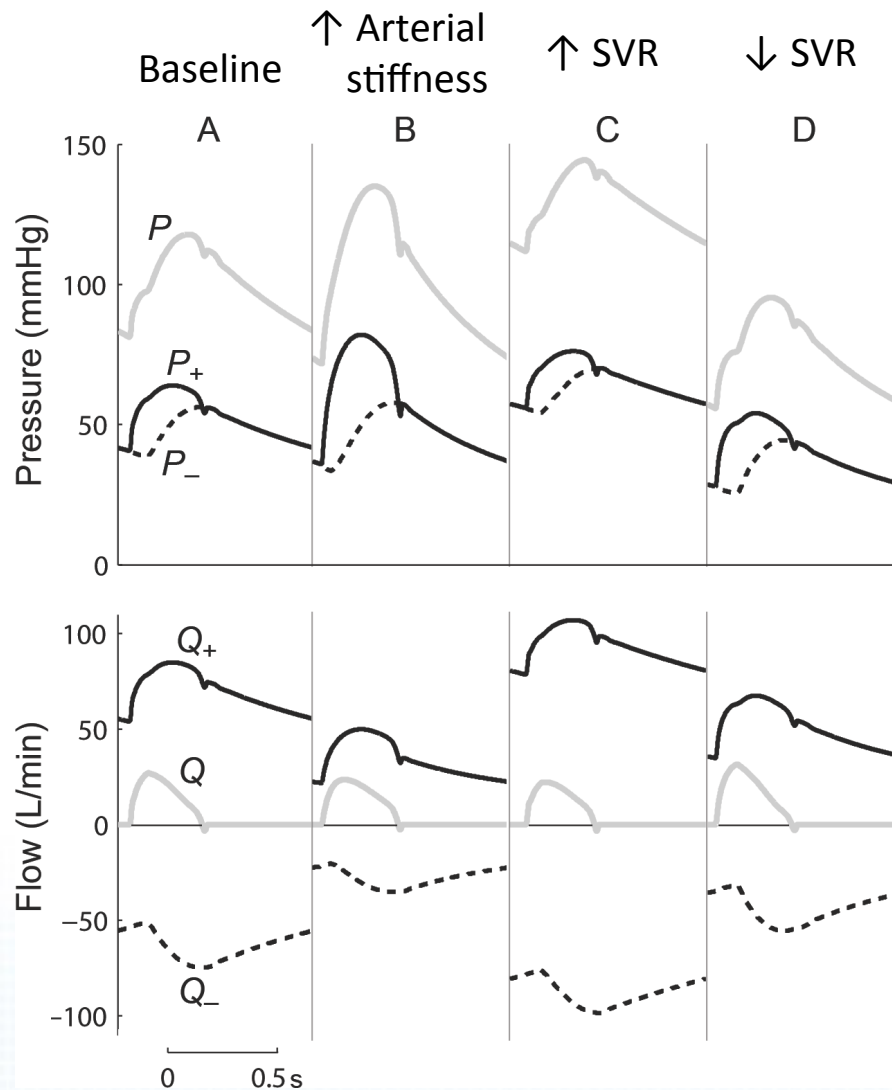
$$P^0 = P_{ud} = 0 \text{ mmHg}$$

$$Q^0 = 0$$



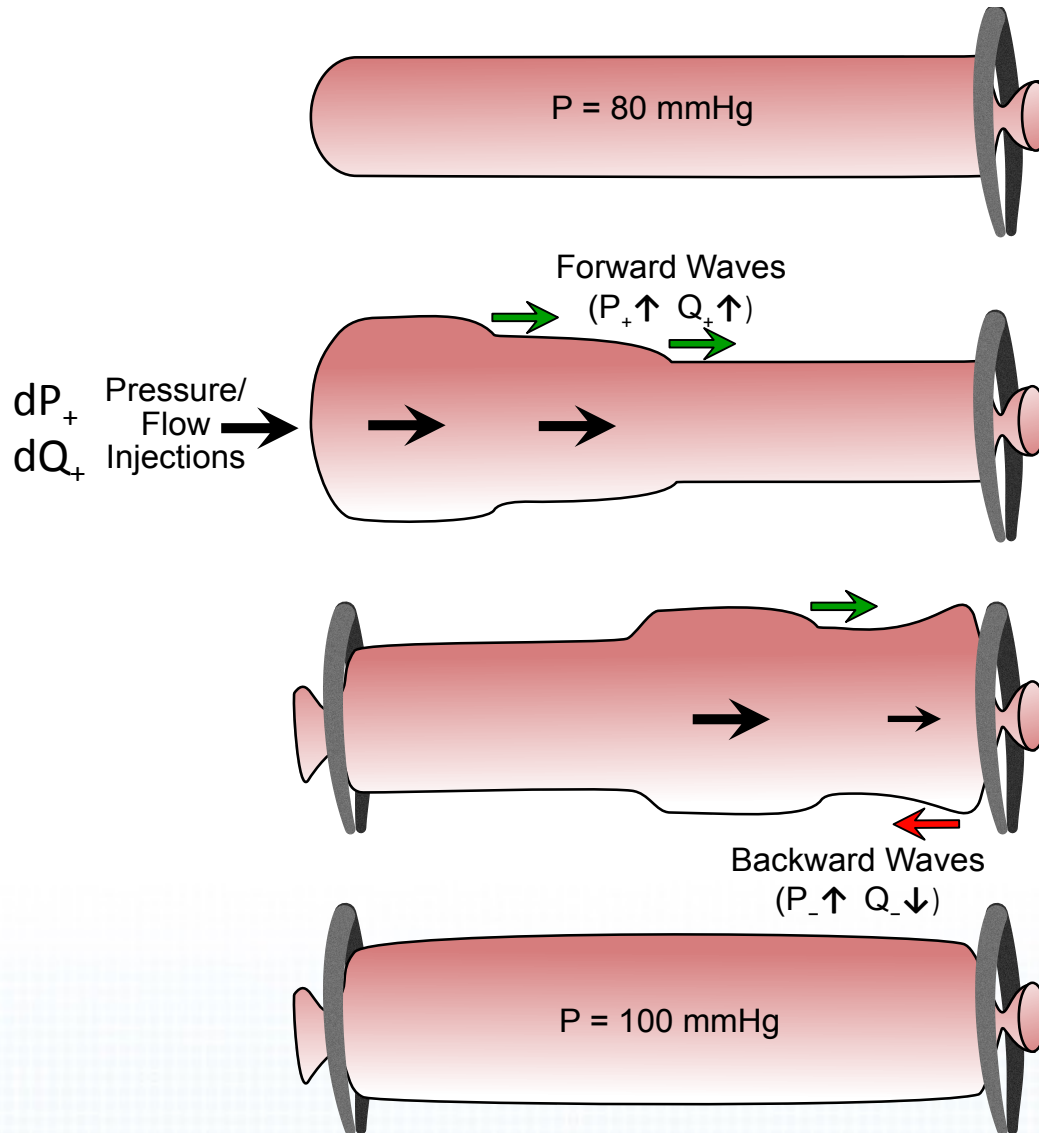
Volume stored/discharged in the first vessel: $V = \int \frac{1}{c} (Q_+^0 - Q_-^0) dx = 5.9 \text{ mL}$

Wave potential and reservoir function



Flow wave potential (Q_{\pm} offset) is an indicator of reservoir function that is directly integrated into wave separation analysis.

Waves and mean pressure



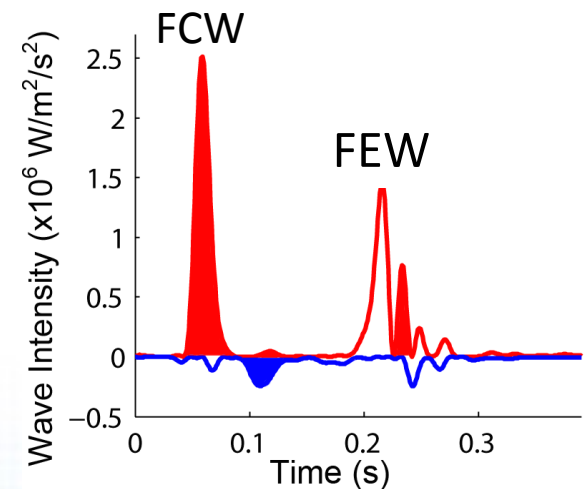
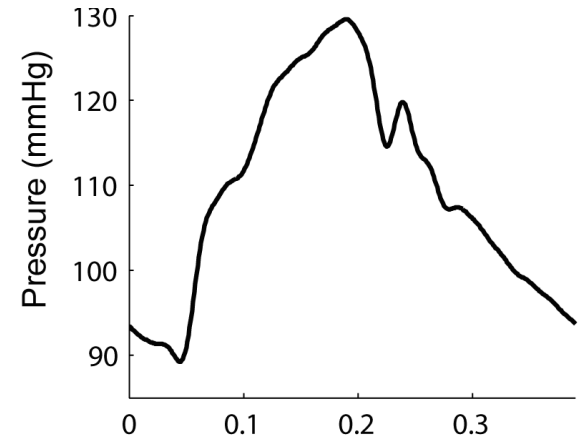
Waves generate and maintain mean pressure

Waves and mean pressure are linked via wave potential

Wave-based explanation of the diastolic pressure decay

Exponential decay is explained by

- distributed/diffuse (re-)reflection of FEW

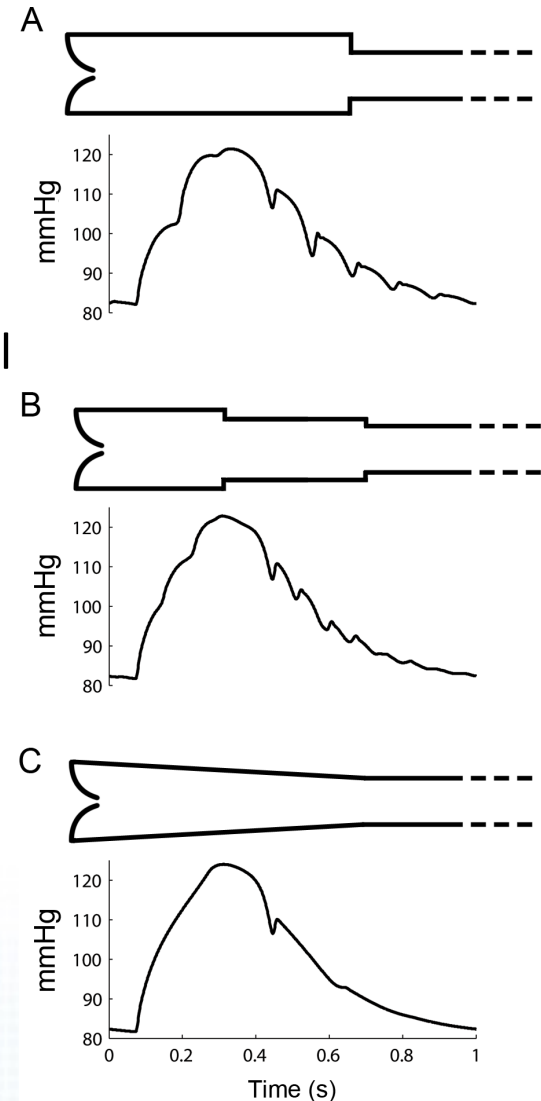


Mynard JP and Smolich JJ. Wave potential and the one-dimensional windkessel as a wave-based paradigm of diastolic arterial hemodynamics. *Am J Physiol Heart Circ Physiol* 307: H307-H318, 2014.

Wave-based explanation of the diastolic pressure decay

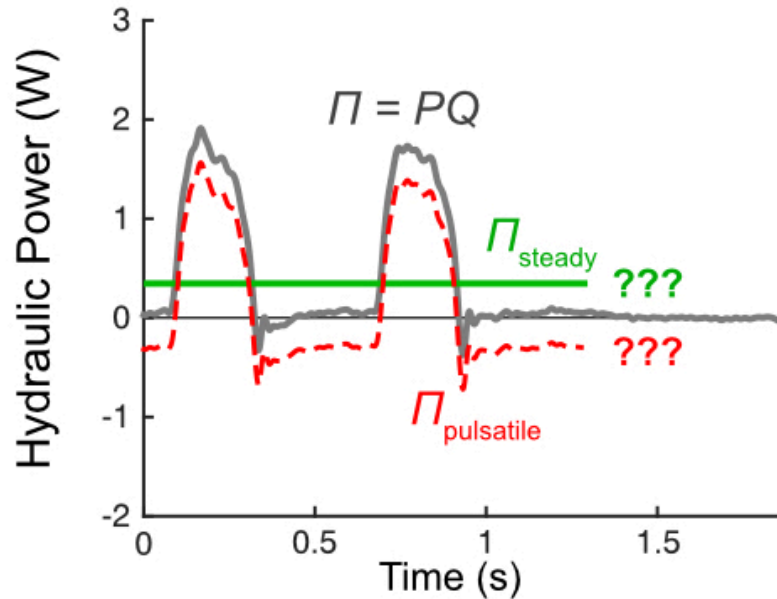
Exponential decay is explained by

- distributed/dispersed (re-)reflection of FEW
- total reflection of P_- at the aortic valve
- distal leakage of blood volume and wave potential



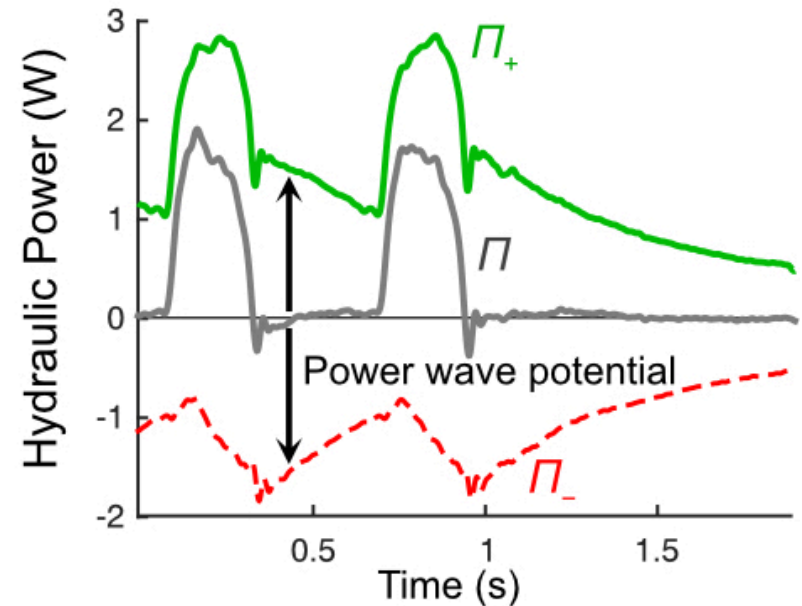
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Wave separation for hydraulic power



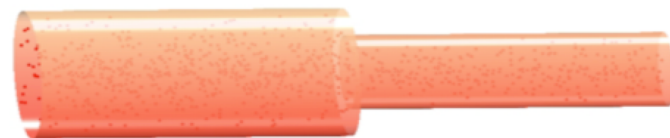
Hydraulic energy potential:

$$E = \int \frac{1}{c} (\Pi_+^0 - \Pi_-^0) dx$$



In the single vessel example:

$$E = 3.75 \text{ mJ}$$



Wave potential - a unified model of haemodynamics

	Wave (Potential) Model	
Mean Pressure	✓	} 'Reservoir' effects
Volume storage	✓	
Exponential P decay	✓	
Pulse propagation	✓	} 'Wave' effects
P Augmentation	✓	
P Amplification	✓	

↑↓

Wave separation for hydraulic power → ventriculo-vascular coupling efficiency?

Thank you!

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- National Health and Medical Research Council of Australia

Acknowledgments

- Joe Smolich
- Patrick Segers
- Berend Westerhof